Solution to Homework 1

A.2. a. The level of GDP per capita in each country, measured in its own currency is

$$(\text{CPUs per capita} \times \text{Price}) + (\text{IC per capita} \times \text{Price}) = \text{GDP per capita.}$$

Therefore, Richland's GDP per capita is 40 and Poorland's GDP per capita is 4.

b. The market exchange rate is determined by the law of one price. As CPUs are the only traded good, the price of computers should be the same. Consequently, the exchange rate must be 2 Richland dollars to 1 Poorland dollar.

c. To find the ratio of GDP per capita between Richland and Poorland, we must first convert GDP denominations into the same currency. In the analysis that follows, I choose to convert GDP denominations into Poorland dollars, but converting to Richland dollars is equally correct, similar, and will yield the same result. From Part (a), we convert Richland's GDP per capita, denominated in Richland dollars, into Poorland dollars by multiplying GDP per capita with the market exchange rate. Since from Part (b), we know 2 Richland dollars equals 1 Poorland dollar, we multiply 1/2 to Richland's GDP per capita, yielding 20 Poorland dollars. Thus, the ratio of Richland GDP per capita to Poorland GDP per capita is 5:1.

d. A natural basket to use is 3 computers and 1 ice cream. The cost of this basket in Richland is 10 Richland dollars. The cost of this basket in Poorland is 4 Poorland dollars. Equating the costs of baskets to be one price, the purchasing power parity exchange rate must be 10 Richland dollars: 4 Poorland dollars.

e. To find the ratio of GDP per capita between Richland and Poorland, we must first convert GDP denominations into the same currency. In the analysis that follows, I choose to convert GDP denominations into Poorland dollars, but converting to Richland dollars is equally correct, similar, and will yield the same result. From Part (a), we convert Richland's GDP per capita, denominated in Richland dollars, into Poorland dollars by multiplying GDP per capita with the PPP exchange rate. Since from Part (d), we know 10 Richland dollars equals 4 Poorland dollars, we multiply 4/10 to Richland's GDP per capita, yielding 16 Poorland dollars. Thus the ratio of Richland GDP per capita to Poorland GDP per capita is 4:1.

6. Simply finding a correlation between being overweight and having a heart attack does not imply causation. The correlation could be due to a missing variable like genetics which may be a factor in a person's weight as well as put him or her at risk for a heart attack. Also, reverse causation may be the reason for the correlation if heart disease has incapacitated a person, thus making him or her unable to exercise which leads to obesity.

2. To find the steady-state value of the country, we refer to Equation (3.3) on page 63.

$$y_{ss} = A^{\frac{1}{1-\alpha}}\left(\frac{\gamma}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Plugging in values: $A = 1$, $\alpha = 0.5$, $\gamma = 0.5$, and $\delta = 0.05$, we get:

$$y_{ss} = 1^{\frac{1}{1-0.5}}\left(\frac{0.5}{0.05}\right)^{\frac{0.5}{1-0.5}}.$$

Simplifying the above equation, we get $y_{ss} = 10$.

To find the current output per worker, we substitute in $k = 400$ into the production function to get:

$$y = k^{\frac{1}{2}} = 400^{\frac{1}{2}} = 20.$$

That is, the current output is 20 whereas the steady-state output level is 10. Therefore, we conclude that $y > y_{ss}$ so the country is above its steady-state level of output per worker.

4. Denoting each variable by the appropriate country subscript, we write Equation (3.3) from page 63 in ratio form. That is,

$$\frac{y_{i,ss}}{y_{j,ss}} = \frac{A_i^{\frac{1}{1-\alpha}}\left(\frac{\gamma_i}{\delta_i}\right)^{\frac{\alpha}{1-\alpha}}}{A_j^{\frac{1}{1-\alpha}}\left(\frac{\gamma_j}{\delta_j}\right)^{\frac{\alpha}{1-\alpha}}}.$$

Since productivity, A, and depreciation, δ, are the same, we can cancel them and rewrite the previous ratio with the appropriate values: $\gamma_1 = 0.05$, $\gamma_2 = 0.2$, and setting $\alpha = 1/3$.

$$\frac{y_{1,ss}}{y_{2,ss}} = \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.05}{0.2}\right)^{\left\{(1/3)/\left(1-\frac{1}{3}\right)\right\}} = (0.25)^{\frac{1}{2}} = 0.5.$$

For $\alpha = 1/2$, we get,

$$\frac{y_{1,ss}}{y_{2,ss}} = \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.05}{0.2}\right)^{\left\{(1/2)/\left(1-\frac{1}{2}\right)\right\}} = (0.25)^{1} = 0.25.$$

Therefore, when $\alpha = 1/3$, the ratio is 0.5 or 1 to 2 and when $\alpha = 1/2$, the ratio is 0.25 or 1 to 4.

7. a. First we find the steady-state level of capital per worker. Using the values for investment, $\gamma = 0.25$, depreciation, $\delta = 0.05$, productivity, $A = 1$, and $\alpha = 0.5$, we get,

$$k_{ss} = \left(\frac{A\gamma}{\delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{(1)(0.25)}{0.05}\right)^{\frac{1}{1-0.5}} = 5^2 = 25.$$

That is, the steady-state level of capital per worker is 25. Plugging in k_{ss} into the production function we get the steady-state level of output per worker to be:

$$y_{ss} = k_{ss}^{\frac{1}{2}} = (25)^{\frac{1}{2}} = 5.$$

That is, the steady-state level of output per worker is 5.

b. For year 2, using 16.2 as the value for capital per worker, calculate output, y, followed by investment γy, depreciation δk, and then change in capital stock. Add the value for change in capital stock to 16.2, the value for capital per worker in year 2, to get capital per worker for year 3. Use year 3 capital to obtain all the values for year 3 and continue up to year 8. The filled in table is below.

Year	Capital	Output	Investment	Depreciation	Change in Capital Stock
1	16.00	4.00	1.00	0.08	0.20
2	16.20	4.02	1.01	0.81	0.20
3	16.40	4.05	1.01	0.82	0.19
4	16.59	4.07	1.02	0.83	0.19
5	16.78	4.10	1.02	0.84	0.19
6	16.96	4.12	1.03	0.85	0.18
7	17.14	4.14	1.04	0.86	0.18
8	17.32	4.16	1.04	0.87	0.17

c. The growth rate of output between years 1 and 2 is given by:

$$g = \left(\frac{y_2}{y_1}\right) - 1 = \left(\frac{4.02}{4}\right) - 1 = 0.005.$$

That is, output per worker grew at a rate of 0.5 percent between years 1 and 2. (Using exact values, the growth rate is approximately 0.62 percent for years 1 and 2.)

d. The growth rate of output between years 7 and 8 is given by:

$$g = \left(\frac{y_8}{y_7}\right) - 1 = \left(\frac{4.16}{4.14}\right) - 1 = 0.0048.$$

That is, output per worker grew at a rate of 0.48 percent between years 7 and 8. (Using exact values, the growth rate is approximately 0.52 percent for years 7 and 8.)

e. The speed of growth has changed from 0.50 percent to 0.48 percent implying that growth has slowed down at a rate of 4 percent. Thus, as a country reaches their steady-state value, the rate of growth slows.